Review Quiz 3

Instructions. You have 15 minutes to complete this review quiz. You may use your calculator. You may not use any other materials. Submit your answers using the provided Google Form.

(1,2) is a critical point

1. If $f_x(1,2) = f_y(1,2) = 0$, $f_{xx}(1,2) = 3$, $f_{yy}(1,2) = 5$, and $f_{xy}(1,2) = 2$, then:

   (a) f has a local minimum at $(1,2)$
   (b) f has a local maximum at $(1,2)$
   (c) f has a saddle point at $(1,2)$
   (d) f has neither a local extreme point nor a saddle point at $(1,2)$
   (e) There is not enough information to determine the behavior of f at $(1,2)$

   $D = f_{xx}f_{yy} - f_{xy}^2 \Rightarrow \left.\begin{array}{l} D(1,2) = 3(5) - 2^2 = 11 > 0 \\ f_{xx}(1,2) = 3 > 0 \end{array}\right\}$ local min at $(1,2)$

2. You want to use Lagrange multipliers to find two positive numbers x and y that add up to 1000 and whose product is maximum. Which of the following systems of equations do you need to solve?

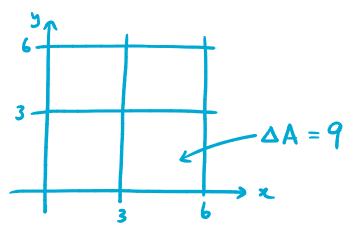
   $\underbrace{x+y}_{g(x,y)} = 1000$ $\qquad$ $f(x,y) = xy$

   (a) $y = \lambda x$, $x = \lambda y$, $x + y = 1000$
   (b) $1000 = \lambda x$, $1000 = \lambda y$, $x + y = 1000$
   (c) $xy = \lambda$, $x + y = \lambda$, $x + y = 1000$
   (d) $y = \lambda(x + y)$, $x = \lambda(x + y)$, $x + y = 1000$
   (e) $y = \lambda$, $x = \lambda$, $x + y = 1000$

   $\nabla f(x,y) = \langle y, x\rangle$
   $\nabla g(x,y) = \langle 1, 1\rangle$ $\Rightarrow$ LM eqs: $\left.\begin{array}{l} y = \lambda \\ x = \lambda \end{array}\right\} \nabla f = \lambda \nabla g$
   $x + y = 1000$

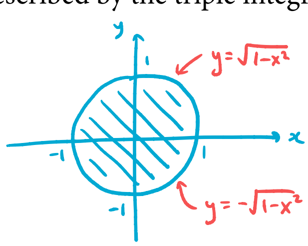
3. We can approximate the double integral $\int_0^6 \int_0^6 f(x,y)\,dy\,dx$ with a Riemann sum by partitioning the region with $0 \le x \le 6$ and $0 \le y \le 6$ into four equal squares. Which expression could arise as our approximation?

   (a) $[f(3,3) + f(3,6) + f(6,3) + f(6,6)] \cdot 4$
   (b) $[f(3,3) + f(3,6) + f(6,3) + f(6,6)] \cdot 6$
   (c) $[f(3,3) + f(3,6) + f(6,3) + f(6,6)] \cdot 9$
   (d) $[f(3,3) + f(3,6) + f(6,3) + f(6,6)] \cdot 16$
   (e) $[f(3,3) + f(3,6) + f(6,3) + f(6,6)] \cdot 36$



4. Which solid has volume described by the triple integral $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^2 1\,dz\,dy\,dx$?

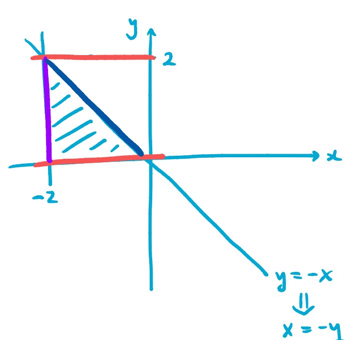
   (a) sphere
   (b) hemisphere
   (c) cone
   (d) cylinder
   (e) cube



   between $z = 0$ and $z = 2$

5. The iterated integral $\int_{-2}^{0} \int_0^{-x} f(x,y)\,dy\,dx$ must be equal to

   (a) $\int_0^{-x} \int_{-2}^{0} f(x,y)\,dx\,dy$
   (b) $\int_0^{2} \int_{-y}^{-2} f(x,y)\,dx\,dy$
   (c) $\int_{-2}^{0} \int_0^{-y} f(x,y)\,dx\,dy$
   (d) $\int_0^{2} \int_{-2}^{-y} f(x,y)\,dx\,dy$
   (e) $\int_{-2}^{0} \int_{-y}^{0} f(x,y)\,dx\,dy$



   $\int_0^2 \int_{-2}^{-y} f(x,y)\,dx\,dy$